Solutionbank M2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.

A particle is projected with speed 35 m s^{-1} at an angle of elevation of $60°$. Find the time the particle takes to reach its greatest height.

Solution:

Resolving the initial velocity vertically

$$R(\uparrow) \quad u_y = 35\sin 60°$$

$$u = 35\sin 60°, v = 0, a = -9.8, t = ?$$

$$v = u + at$$

$$0 = 35\sin 60° - 9.8t$$

$$t = \frac{35\sin 60°}{9.8} = 3.092\ldots \approx 3.1$$

The time the particle takes to reach its greatest height is 3.1 (2 s.f.).

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Exercise A, Question 2

Question:

A ball is projected from a point 5 m above horizontal ground with speed 18 m s^{-1} at an angle of elevation of $40°$. Find the height of the ball above the ground 2 s after projection.

Solution:

Resolving the initial velocity vertically

$$\text{R}(\uparrow) \quad u_y = 18\sin 40°$$

$$u = 18\sin 40°, a = -9.8, t = 2, s = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$= 18\sin 40° \times 2 - 4.9 \times 2^2$$

$$= 3.540\ldots \approx 3.5$$

The height of the ball above the ground 2 s after projection is $(5 + 3.5)\text{ m} = 8.5\text{ m}$ (2 s.f.).

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Exercise A, Question 3

Question:

A stone is projected horizontally from a point above horizontal ground with speed 32 m s^{-1}. The stone takes 2.5 s to reach the ground. Find

a the height of the point of projection above the ground,

b the distance from the point on the ground vertically below the point of projection to the point where the stone reached the ground.

Solution:

Resolving the initial velocity horizontally and vertically

$$\text{R}(\rightarrow) \quad u_x = 32$$
$$\text{R}(\downarrow) \quad u_y = 0$$

a

$$\text{R}(\downarrow) \quad u = 0, a = 9.8, t = 2.5, s = ?$$
$$s = ut + \frac{1}{2}at^2$$
$$= 0 + 4.9 \times 2.5^2 = 30.625 \approx 31$$

The height of the point of projection above the ground is 31 m (2 s.f.).

b

$$\text{R}(\rightarrow) \quad \text{distance} = \text{speed} \times \text{time}$$
$$= 32 \times 2.5 = 80$$

The horizontal distance moved is 80 m.

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Exercise A, Question 4

Question:

A projectile is launched from a point on horizontal ground with speed 150 m s^{-1} at an angle of $10°$ to the horizontal. Find

a the time the projective takes to reach its highest point above the ground,

b the range of the projectile.

Solution:

Resolving the initial velocity horizontally and vertically

$$R(\rightarrow) \quad u_x = 150\cos 10°$$
$$R(\uparrow) \quad u_y = 150\sin 10°$$

a

$$R(\uparrow) \quad u = 150\sin 10°, v = 0, a = -9.8, t = ?$$
$$v = u + at$$
$$0 = 150\sin 10° - 9.8t$$
$$t = \frac{150\sin 10°}{9.8} = 2.657\ldots \approx 2.7$$

The time taken to reach the projectile's highest point is 2.7 s (2 s.f.).

b By symmetry, the time of flight is $(2.657\ldots \times 2)\text{s} = 5.315\ldots \text{s}$.

$$R(\rightarrow) \quad \text{distance} = \text{speed} \times \text{time}$$
$$= 150\cos 10° \times 5.315\ldots$$
$$= 785.250\ldots \approx 790$$

The range of the projectile is 790 m (2 s.f.).

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Exercise A, Question 5

Question:

A particle is projected from a point O on a horizontal plane with speed 20 m s^{-1} at an angle of elevation of $45°$. The particle moves freely under gravity until it strikes the ground at a point X. Find

a the greatest height above the plane reached by the particle,

b the distance OX.

Solution:

Resolving the initial velocity horizontally and vertically

$$R(\rightarrow) \quad u_x = 20\cos 45° = 10\sqrt{2}$$
$$R(\uparrow) \quad u_y = 20\sin 45° = 10\sqrt{2}$$

a

$$R(\uparrow) \quad u = 10\sqrt{2}, v = 0, a = -9.8, s = ?$$
$$v^2 = u^2 + 2as$$
$$0 = 200 - 19.6s$$
$$s = \frac{200}{19.6} = 10.204\ldots \approx 10$$

The greatest height above the plane reached by the particle is 10 m (2 s.f.).

b To find the time taken to move from O to X

$$R(\uparrow) \quad s = 0, u = 10\sqrt{2}, a = -9.8, t = ?$$
$$s = ut + \frac{1}{2}at^2$$
$$0 = 10\sqrt{2}t - 4.9t^2 = t(10\sqrt{2} - 4.9t)$$
($t = 0$ corresponds to the point of projection.)
$$t = \frac{10\sqrt{2}}{4.9} = 2.886\ldots$$

$$R(\rightarrow) \quad \text{distance} = \text{speed} \times \text{time}$$
$$= 10\sqrt{2} \times 2.886\ldots = 40.816\ldots \approx 41$$
$$OX = 41\text{ m} \quad (2\text{ s.f.})$$

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Exercise A, Question 6

Question:

A ball is projected from a point A on level ground with speed 24 m s^{-1}. The ball is projected at an angle θ to the horizontal where $\sin\theta = \frac{4}{5}$. The ball moves freely under gravity until it strikes the ground at a point B. Find

a the time of flight of the ball,

b the distance from A to B.

Solution:

$\sin\theta = \frac{4}{5} \Rightarrow \cos\theta = \frac{3}{5}$

Resolving the initial velocity horizontally and vertically

$\text{R}(\rightarrow) \quad u_x = 24\cos\theta = 14.4$

$\text{R}(\uparrow) \quad u_y = 24\sin\theta = 19.2$

a

$\text{R}(\uparrow) \quad u = 19.2, s = 0, a = -9.8, t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 19.2t - 4.9t^2 = t(19.2 - 4.9t)$

($t = 0$ corresponds to the point of projection.)

$t = \frac{19.2}{4.9} = 3.918\ldots \approx 3.9$

The time of flight of the ball is 3.9 s (2 s.f.)

b

$\text{R}(\rightarrow) \quad \text{distance} = \text{speed} \times \text{time}$

$= 14.4 \times 3.918\ldots = 56.424\ldots \approx 56$

$AB = 56 \text{ m} \quad (2 \text{ s.f.})$

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Exercise A, Question 7

Question:

A particle is projected with speed 21 m s^{-1} at an angle of elevation α. Given that the greatest height reached above the point of projection is 15 m, find the value of α, giving your answer to the nearest degree.

Solution:

Resolving the initial velocity vertically and angle of elevation $= \alpha$

$$R(\uparrow) \quad u_y = 21 \sin \alpha$$

$$u = 21 \sin \alpha, v = 0, s = 15, a = -9.8$$

$$v^2 = u^2 + 2as$$

$$0 = (21 \sin \alpha)^2 - 2 \times 9.8 \times 15$$

$$441 \sin^2 \alpha = 294$$

$$\sin^2 \alpha = \frac{294}{441} = \frac{2}{3} \Rightarrow \sin \alpha = \sqrt{\left(\frac{2}{3}\right)} = 0.816\ldots$$

$$\alpha \approx 54.736° \approx 55° \text{ (nearest degree)}$$

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Exercise A, Question 8

Question:

A particle is projected horizontally from a point A which is 16 m above horizontal ground. The projectile strikes the ground at a point B which is at a horizontal distance of 140 m from A. Find the speed of projection of the particle.

Solution:

$R(\downarrow)\quad u = 0, s = 16, a = 9.8, t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$16 = 0 + 4.9t^2$$

$$t^2 = \frac{16}{4.9} = 3.265\ldots \Rightarrow t = 1.807\ldots$$

Let the speed of projection be u m s^{-1}.

$R(\rightarrow)\quad \text{distance} = \text{speed} \times \text{time}$

$$140 = u \times 1.807\ldots$$

$$u = \frac{140}{1.807\ldots} = 77.475\ldots \approx 77$$

The speed of projection of the particle is 77 m s^{-1} (2 s.f.).

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Exercise A, Question 9

Question:

A particle P is projected from the origin with velocity $(12\mathbf{i} + 24\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical unit vectors respectively. The particle moves freely under gravity. Find

a the position vector of P after 3 s,

b the speed of P after 3 s.

Solution:

a

R($\rightarrow$) distance = speed × time

$= 12 \times 3 = 36$

R($\uparrow$) $s = ut + \frac{1}{2}at^2$

$= 24 \times 3 - 4.9 \times 9 = 27.9$

The position vector of P after 3 s is $(36\mathbf{i} + 27.9\mathbf{j})$m .

b

R($\rightarrow$) $u_x = 12$, throughout the motion

R($\uparrow$) $v = u + at$

$v_y = 24 - 9.8 \times 3 = -5.4$

Let the speed of P after 3 s be V m s^{-1}.

$V^2 = u_x^2 + v_y^2 = 12^2 + (-5.4)^2 = 173.16$

$V = \sqrt{173.16} = 13.159\ldots \approx 13$

The speed of P after 3 s is 13 m s^{-1} (2 s.f.).

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Exercise A, Question 10

Question:

A stone is thrown with speed 30 m s^{-1} from a window which is 20 m above horizontal ground. The stone hits the ground 3.5 s later. Find

a the angle of projection of the stone,

b the horizontal distance from the window to the point where the stone hits the ground.

Solution:

Let α be the angle of projection above the horizontal.

Resolving the initial velocity horizontally and vertically

$$R(\rightarrow) \quad u_x = 30\cos\alpha$$
$$R(\uparrow) \quad u_y = 30\sin\alpha$$

a

$$R(\uparrow) \quad u = 30\sin\alpha, s = -20, a = -9.8, t = 3.5$$
$$s = ut + \frac{1}{2}at^2$$
$$-20 = 30\sin\alpha \times 3.5 - 4.9 \times 3.5^2$$
$$\sin\alpha = \frac{4.9 \times 3.5^2 - 20}{30 \times 3.5} = 0.381190\ldots$$
$$\alpha = 22.407\ldots^\circ = 22^\circ$$

The angle of projection of the stone is 22° (2 s.f.) above the horizontal.

b

$$R(\rightarrow) \quad \text{distance} = \text{speed} \times \text{time}$$
$$= 30\cos\alpha \times 3.5 = 97.072\ldots$$

The horizontal distance from the window to the point where the stone hits the ground is 97 m (2 s.f.).

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Exercise A, Question 11

Question:

A ball is thrown from a point O on horizontal ground with speed u m s^{-1} at an angle of elevation of θ, where $\tan\theta = \frac{3}{4}$. The ball strikes a vertical wall which is 20 m from O at a point which is 3 m above the ground. Find

a the value of u,

b the time from the instant the ball is thrown to the instant that it strikes the wall.

Solution:

$\tan\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}$

Resolving the initial velocity horizontally and vertically

$$\text{R}(\rightarrow) \quad u_x = u\cos\theta = \frac{4u}{5}$$

$$\text{R}(\uparrow) \quad u_y = u\sin\theta = \frac{3u}{5}$$

a

$$\text{R}(\rightarrow) \quad \text{distance} = \text{speed} \times \text{time}$$

$$20 = \frac{4u}{5} \times t \Rightarrow t = \frac{25}{u}$$

$$\text{R}(\uparrow) \quad s = ut + \frac{1}{2}at^2$$

$$3 = \frac{3u}{5}t - 4.9t^2 \qquad (1)$$

Substituting $t = \frac{25}{u}$ into (1)

$$3 = \frac{3u}{5} \times \frac{25}{u} - 4.9 \times \frac{25^2}{u^2}$$

$$3 = 15 - \frac{3062.5}{u^2} \Rightarrow u^2 = \frac{3062.5}{12} = 255.208\ldots$$

$$u = \sqrt{255.208\ldots} = 15.975\ldots \approx 16$$

$$u = 16 \text{ (2 s.f.)}$$

b $t = \frac{25}{u} = \frac{25}{15.975\ldots} = 1.5649\ldots \approx 1.6$

The time from the instant the ball is thrown to the instant that it strikes the wall is 1.6 s (2 s.f.).

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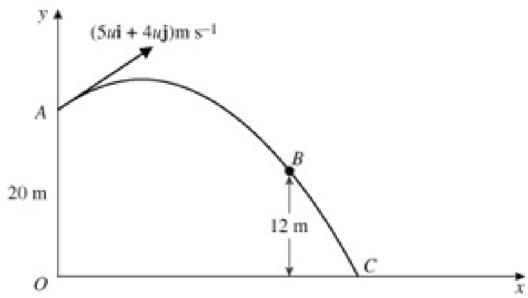
Exercise A, Question 12

Question:



[*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertical.*]

A particle P is projected from a point A with position vector $20\mathbf{j}$ m with respect to a fixed origin O. The velocity of projection is $(5u\mathbf{i} + 4u\mathbf{j})$ m s^{-1}. The particle moves freely under gravity, passing through a point B, which has position vector $(k\mathbf{i} + 12\mathbf{j})$ m, where k is a constant, before reaching the point C on the x-axis, as shown in the figure above. The particle takes 4 s to move from A to B. Find

a the value of u,

b the value of k,

c the angle the velocity of P makes with the x-axis as it reaches C.

Solution:

a

$R(\uparrow) \quad s = ut + \frac{1}{2}at^2$

$-8 = 4u \times 4 - 4.9 \times 4^2$

$u = \frac{4.9 \times 4^2 - 8}{16} = 4.4$

b

$R(\rightarrow) \quad \text{distance} = \text{speed} \times \text{time}$

$k = 5u \times t = 5 \times 4.4 \times 4 = 88$

c $u_x = 5u = 5 \times 4.4 = 22$, throughout the motion.

At C

$R(\uparrow) \quad v^2 = u^2 + 2as$

$v_y^2 = (4u)^2 + 2 \times (-9.8) \times (-20)$

$= 16 \times 4.4^2 + 392 = 701.76$

Let θ be angle the velocity of P makes with Ox as it reaches C.

$\tan\theta = \frac{v_y}{u_x} = \frac{\sqrt{701.76}}{22} = 1.204\ldots$

$\theta = 50.129\ldots \approx 50^\circ$

The angle the velocity of P makes with Ox as it reaches C is 50° (2 s.f.).

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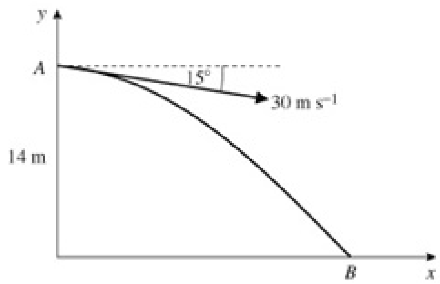
Exercise A, Question 13

Question:



A stone is thrown from a point A with speed 30 m s^{-1} at an angle of $15°$ below the horizontal. The point A is 14 m above horizontal ground. The stone strikes the ground at the point B, as shown in the figure above. Find

a the time the stone takes to travel from A to B,

b the distance AB.

Solution:

Resolving the initial velocity horizontally and vertically

$R(\rightarrow) \quad u_x = 30\cos 15°$

$R(\downarrow) \quad u_y = 30\sin 15°$

a

$R(\downarrow) \quad u = 30\sin 15°, s = 14, a = 9.8, t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$14 = 30\sin 15° t + 4.9t^2$$

$$4.9t^2 + 30\sin 15° t - 14 = 0$$

Using the formula for solving the quadratic, (the negative solution can be ignored)

$$t = \frac{-30\sin 15° + \sqrt{(900\sin^2 15 + 4\times 14\times 4.9)}}{9.8}$$

$$= 1.074\ldots \approx 1.1$$

The time the particle takes to travel from A to B is 1.1 s (2 s.f.)

b

$R(\rightarrow) \quad \text{distance} = \text{speed} \times \text{time}$

$$= 30\cos 15° \times 1.074\ldots$$

$$= 31.136\ldots$$

$$AB^2 = 14^2 + (31.136\ldots)^2 = 1165.196\ldots$$

$$AB = 34.138\ldots \approx 34$$

The distance AB is 34 m (2 s.f).

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Exercise A, Question 14

Question:

A particle is projected from a point with speed 21 m s^{-1} at an angle of elevation α and moves freely under gravity. When the particle has moved a horizontal distance x m, its height above the point of projection is y m.

a Show that $y = x\tan\alpha - \dfrac{x^2}{90\cos^2\alpha}$.

b Given that $y = 8.1$ when $x = 36$, find the value of $\tan\alpha$.

Solution:

Resolving the initial velocity horizontally and vertically

$R(\rightarrow)\quad u_x = 21\cos\alpha$

$R(\uparrow)\quad u_y = 21\sin\alpha$

a $R(\rightarrow)$ distance = speed × time

$$x = 21\cos\alpha \times t \Rightarrow t = \frac{x}{21\cos\alpha}$$

$R(\uparrow)\quad s = ut + \frac{1}{2}at^2$

$$y = 21\sin\alpha t - \frac{g}{2}t^2$$

$$= 21\sin\alpha\left(\frac{x}{21\cos\alpha}\right) - 4.9\left(\frac{x}{21\cos\alpha}\right)^2$$

$$= x\tan\alpha - \frac{4.9x^2}{441\cos^2\alpha} = x\tan\alpha - \frac{x^2}{90\cos^2\alpha}\text{, as required}$$

b $\dfrac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$

Using $y = 8.1, x = 36$ and the result in **a**

$$8.1 = 36\tan\alpha - \frac{36^2}{90}(1 + \tan^2\alpha) = 36\tan\alpha - 14.4(1 + \tan^2\alpha)$$

$\times 10$ and rearranging

$$144\tan^2\alpha - 360\tan\alpha + 225 = 0$$

$$(\div 9)\ 16\tan^2\alpha - 40\tan\alpha + 25 = (4\tan\alpha - 5)^2 = 0$$

$$\tan\alpha = \frac{5}{4}$$

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Exercise A, Question 15

Question:

A projectile is launched from a point on a horizontal plane with initial speed u m s^{-1} at an angle of elevation α. The particle moves freely under gravity until it strikes the plane. The range of the projectile is R m.

a Show that the time of flight of the particle is $\frac{2u\sin\alpha}{g}$ seconds.

b Show that $R = \frac{u^2 \sin 2\alpha}{g}$.

c Deduce that, for a fixed u, the greatest possible range is when $\alpha = 45°$.

d Given that $R = \frac{2u^2}{5g}$, find the two possible values of the angle of elevation at which the projectile could have been launched.

Solution:

Resolving the initial velocity horizontally and vertically

R(→) $u_x = u\cos\alpha$

R(↑) $u_y = u\sin\alpha$

a

R(↑) $s = ut + \frac{1}{2}at^2$

$$0 = u\sin\alpha t - \frac{1}{2}gt^2 = t\left(u\sin\alpha - \frac{1}{2}gt\right)$$

($t = 0$ corresponds to the point of projection.)

$\frac{1}{2}gt = u\sin\alpha \Rightarrow t = \frac{2u\sin\alpha}{g}$, as required

b

R(→) distance = speed × time

$$R = u\cos\alpha \times \frac{2u\sin\alpha}{g} = \frac{u^2 \times 2\sin\alpha\cos\alpha}{g}$$

Using the trigonometric identity $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$R = \frac{u^2\sin 2\alpha}{g}$, as required

c The greatest possible value of $\sin 2\alpha$ is 1, which is when $2\alpha = 90° \Rightarrow \alpha = 45°$.

Hence, for a fixed u, the greatest possible range is when $\alpha = 45°$.

d

$$\frac{2u^2}{5g} = \frac{u^2\sin 2\alpha}{g} \Rightarrow \sin 2\alpha = \frac{2}{5}$$

$2\alpha \approx 23.578°, 156.422°$

$\alpha \approx 11.79°, 78.21°$

The two possible angles of elevation are $12°$ and $78°$ (nearest degree).

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Exercise A, Question 16

Question:

A particle is projected from a point on level ground with speed u m s^{-1} and angle of elevation α. The maximum height reached by the particle is 42 m above the ground and the particle hits the ground 196 m from its point of projection.

Find the value of α and the value of u.

Solution:

Resolving the initial velocity horizontally and vertically

$$\text{R}(\rightarrow) \quad u_x = u\cos\alpha$$
$$\text{R}(\uparrow) \quad u_y = u\sin\alpha$$

Using the maximum height is 42 m

$$\text{R}(\uparrow) \quad v^2 = u^2 + 2as$$
$$0 = u^2\sin^2\alpha - 2g \times 42$$
$$u^2\sin^2\alpha = 84g \qquad (1)$$

For the range

$$\text{R}(\rightarrow) \quad \text{distance} = \text{speed} \times \text{time}$$
$$196 = u\cos\alpha \times t \Rightarrow t = \frac{196}{u\cos\alpha} \qquad (2)$$
$$\text{R}(\uparrow) \quad s = ut + \frac{1}{2}at^2$$
$$0 = u\sin\alpha t - \frac{1}{2}gt^2 = t\left(u\sin\alpha - \frac{1}{2}gt\right)$$
$$\frac{1}{2}gt = u\sin\alpha \Rightarrow t = \frac{2u\sin\alpha}{g} \qquad (3)$$

From (2) and (3)

$$\frac{196}{u\cos\alpha} = \frac{2u\sin\alpha}{g}$$
$$u^2\sin\alpha\cos\alpha = 98g \qquad (4)$$

Dividing (1) by (4)

$$\frac{u^2\sin^2\alpha}{u^2\sin\alpha\cos\alpha} = \frac{84g}{98g}$$
$$\tan\alpha = \frac{6}{7} \Rightarrow \alpha = 40.6^\circ \text{ (nearest } 0.1^\circ\text{)}$$

From (1)

$$u\sin\alpha = \sqrt{(84g)}$$
$$u = \frac{\sqrt{(84 \times 9.8)}}{\sin 40.6^\circ} = 44.08\ldots = 44 \text{ (2 s.f.)}$$

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Exercise B, Question 1

Question:

A particle is moving in a straight line. At time t seconds, its displacement, x m, from a fixed point O on the line is given by $x = 2t^3 - 8t$. Find

a the speed of the particle when $t = 3$,

b the magnitude of the acceleration of the particle when $t = 2$.

Solution:

a

$$x = 2t^3 - 8t$$

$$v = \frac{dx}{dt} = 6t^2 - 8$$

When $t = 3$

$$v = 6 \times 3^2 - 8 = 46$$

The speed of the particle when $t = 3$ is 46 m s^{-1}.

b $a = \frac{dv}{dt} = 12t$

When $t = 2$,

$$a = 12 \times 2 = 24$$

The magnitude of the acceleration of the particle when $t = 2$ is 24 m s^{-2}.

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Exercise B, Question 2

Question:

A particle P is moving on the x-axis. At time t seconds, the velocity of P is $(8+2t-3t^2)$ m s^{-1} in the direction of x increasing. At time $t=0$, P is at the point where $x=4$. Find

a the magnitude of the acceleration of P when $t=3$,

b the distance of P from O when $t=1$.

Solution:

a

$$v = 8+2t-3t^2$$
$$a = \frac{dv}{dt} = 2-6t$$

When $t=3$,

$2-6\times3=-16$

The magnitude of the acceleration of P when $t=3$ is 16 m s^{-2}.

b

$$x = \int v\,dt$$
$$= 8t+t^2-t^3+c, \text{ where } c \text{ is a constant of integration.}$$

When $t=0, x=4$

$$4 = 0+0-0+c \Rightarrow c=4$$
$$x = 4+8t+t^2-t^3$$

When $t=1$,

$$x = 4+8+1-1=12$$

The distance of P from O when $t=1$ is 12 m.

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Exercise B, Question 3

Question:

A particle P is moving on the x-axis. At time t seconds, the acceleration of P is $(16-2t)$ m s^{-2} in the direction of x increasing. The velocity of P at time t seconds is v m s^{-1}.

When $t=0, v=6$ and when $t=3, x=75$. Find

a v in terms of t,

b the value of x when $t=0$.

Solution:

a

$v=\int a\,dt$

$=16t-t^2+c$, where c is a constant of integration.

When $t=0, v=6$

$6=0-0+c \Rightarrow c=6$

$v=6+16t-t^2$

b

$x=\int v\,dt$

$=6t+8t^2-\frac{t^3}{3}+k$, where k is a constant of integration.

When $t=3, x=75$

$75=6\times3+8\times9-\frac{27}{3}+k$

$k=75-18-72+9=-6$

$x=6t+8t^2-\frac{t^3}{3}-6$

When $t=0$,

$x=0+0-0-6=-6$

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Exercise B, Question 4

Question:

A particle P is moving on the x-axis. At time t seconds (where $t \geq 0$), the velocity of P is v m s^{-1} in the direction of x increasing, where $v = 12 - t - t^2$.

Find the acceleration of P when P is instantaneously at rest.

Solution:

P is at rest when $v = 0$

$$0 = 12 - t - t^2$$

$$t^2 + t - 12 = (t+4)(t-3) = 0$$

$$t = -4, 3$$

As $t \geq 0, t = -4$ is rejected.

$$a = \frac{dv}{dt} = -1 - 2t$$

When $t = 3$,

$$a = -1 - 2 \times 3 = -7$$

The acceleration of P when P comes to instantaneously to rest is 7 m s^{-2} in the direction of x decreasing.

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Exercise B, Question 5

Question:

A particle is moving in a straight line. At time t seconds, its displacement, x m, from a fixed point O on the line is given by $x = 4t^3 - 39t^2 + 120t$.

Find the distance between the two points where P is instantaneously at rest.

Solution:

$x = 4t^3 - 39t^2 + 120t$

$v = \frac{dx}{dt} = 12t^2 - 78t + 120$

P is at rest when $v = 0$

$12t^2 - 78t + 120 = 6(2t^2 - 13t + 20) = 6(2t - 5)(t - 4) = 0$

$t = 2.5, 4$

When $t = 2.5$,

$x = 4(2.5)^3 - 39(2.5)^2 + 120 \times 2.5 = 118.75$

When $t = 4$,

$x = 4(4)^3 - 39(4)^2 + 120 \times 4 = 112$

The distance between the two points where P is instantaneously at rest is

$(118.75 - 112)\text{m} = 6.75\text{ m}.$

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 6

Question:

At time t seconds, where $t \geq 0$, the velocity v m s^{-1} of a particle moving in a straight line is given by $v = 12 + t - 6t^2$. When $t = 0$, P is at a point O on the line. Find

a the magnitude of the acceleration of P when $v = 0$,

b the distance of P from O when $v = 0$.

Solution:

a When $v = 0$,

$$12 + t - 6t^2 = 0$$
$$6t^2 - t - 12 = (2t - 3)(3t + 4) = 0$$
$$t = \frac{3}{2}, -\frac{4}{3}$$

As $t \geq 0, t = -\frac{4}{3}$ is rejected.

$$a = \frac{dv}{dt} = 1 - 12t$$

When $t = \frac{3}{2}$,

$$a = 1 - 12 \times \frac{3}{2} = -17$$

The magnitude of the acceleration of P when $v = 0$ is 17 m s^{-2}.

b

$$x = \int v \, dt$$
$$= 12t + \frac{1}{2}t^2 - \frac{6}{3}t^3 + c, \text{ where } c \text{ is a constant of integration.}$$

When $t = 0, x = 0$

$$0 = 0 + 0 - 0 + c \Rightarrow c = 0$$

When $t = \frac{3}{2}$,

$$x = 12 \times 1.5 + \frac{1.5^2}{2} - 2 \times 1.5^3 = 12.375$$

The distance of P from O when $v = 0$ is 12.375 m.

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 7

Question:

A particle P is moving on the x-axis. At time t seconds, the velocity of P is $(4t - t^2)$ m s^{-1} in the direction of x increasing. At time $t = 0$, P is at the origin O. Find

a the value of x at the instant when $t > 0$ and P is at rest,

b the total distance moved by P in the interval $0 \le t \le 5$.

Solution:

a P is at rest when $v = 0$

$$v = 4t - t^2 = 0$$
$$t(4 - t) = 0$$

As $t > 0$, $t = 4$

$$x = \int v \, \mathrm{d}t$$
$$= 2t^2 - \frac{1}{3}t^2 + c$$

When $t = 0, x = 0$

$$0 = 0 - 0 + c = 0 \Rightarrow c = 0$$
$$x = 2t^2 - \frac{1}{3}t^3$$

When $t = 4$

$$x = 2 \times 4^2 - \frac{4^3}{3} = 10\frac{2}{3}$$

b When $t = 5$,

$$x = 2 \times 5^2 - \frac{5^3}{3} = 8\frac{1}{3}$$

In the interval $0 \le t \le 5$, moves to a point $10\frac{2}{3}$ m from O and then returns to a point $8\frac{1}{3}$ m from O.

The total distance moved is $10\frac{2}{3} + \left(10\frac{2}{3} - 8\frac{1}{3}\right) = 13$ m.

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Exercise B, Question 8

Question:

A particle P is moving on the x-axis. At time t seconds, the velocity of P is $(6t^2 - 26t + 15)$ m s^{-1} in the direction of x increasing. At time $t = 0$, P is at the origin O. In the subsequent motion P passes through O twice. Find

a the two non-zero values of t when P passes through O,

b the acceleration of P for these two values of t.

Solution:

a

$x = \int v \, dt$

$= 2t^3 - 13t^2 + 15t + c$, where c is a constant of integration.

When $t = 0, x = 0$

$0 = 0 - 0 + 0 + c \Rightarrow c = 0$

$x = 2t^3 - 13t^2 + 15t = t(2t - 3)(t - 5)$

When $x = 0$ and t is non-zero

$t = \frac{3}{2}, 5$

b

$a = \frac{dv}{dt} = 12t - 26$

When $t = \frac{3}{2}, a = 12 \times \frac{3}{2} - 26 = -8$

The acceleration of P is 8 m s^{-2} in the direction of x decreasing.

When $t = 5, a = 12 \times 5 - 26 = 34$

Then acceleration of P is 34 m s^{-2} in the direction of x increasing.

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Exercise B, Question 9

Question:

A particle P of mass 0.4 kg is moving in a straight line under the action of a single variable force $\mathbf{F}$ newtons. At time t seconds (where $t \geq 0$) the displacement x m of P from a fixed point O is given by $x = 2t + \frac{k}{t+1}$, where k is a constant. Given that when $t = 0$, the velocity of P is 6 m s^{-1}, find

a the value of k,

b the distance of P from O when $t = 0$,

c the magnitude of $\mathbf{F}$ when $t = 3$.

Solution:

a

$x = 2t + k(t+1)^{-1}$

$v = \frac{dx}{dt} = 2 - k(t+1)^{-2} = 2 - \frac{k}{(t+1)^2}$

When $t = 0, v = 6$

$6 = 2 - \frac{k}{1^2} \Rightarrow k = -4$

b With $k = -4$,

$x = 2t - \frac{4}{t+1}$

When $t = 0$,

$x = 0 - \frac{4}{0+1} = -4$

The distance of P from O when $t = 0$ is 4 m.

c

$v = 2 - 4(t+1)^{-2}$

$a = \frac{dv}{dt} = 8(t+1)^{-3} = \frac{8}{(t+1)^3}$

When $t = 3$

$a = \frac{8}{4^3} = \frac{1}{8}$

$F = ma$

$= 0.4 \times \frac{1}{8} = 0.05$

The magnitude of $\mathbf{F}$ when $t = 3$ is 0.05.

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 10

Question:

A particle P moves along the x-axis. At time t seconds (where $t \geq 0$) the velocity of P is $(3t^2 - 12t + 5)$ m s^{-1} in the direction of x increasing. When $t = 0$, P is at the origin O. Find

a the velocity of P when its acceleration is zero,

b the values of t when P is again at O,

c the distance travelled by P in the interval $3 \leq t \leq 4$.

Solution:

a $a = \frac{dv}{dt} = 6t - 12 = 0 \Rightarrow t = 2$

When $t = 2$,

$v = 3 \times 2^2 - 12 \times 2 + 5 = -7$

The velocity of P when the acceleration is zero is 7 m s^{-1} in the direction of x decreasing.

b

$$s = \int (3t^2 - 12t + 5)\,dt$$
$$= t^3 - 6t^2 + 5t + C$$

When $t = 0, s = 0$

$$0 = 0 - 0 + 0 + C \Rightarrow C = 0$$
$$s = t^3 - 6t^2 + 5t$$

P returns to O when $s = 0$

$$s = t^3 - 6t^2 + 5t = t(t-1)(t-5) = 0$$
$$t = 1, 5$$

c When $t = 3, s = 3^3 - 6 \times 3^2 + 5 \times 3 = -12$

When $t = 4, s = 4^3 - 6 \times 4^2 + 5 \times 4 = -60$

The distance travelled by P in the interval $3 \leq t \leq 4$ is 48 m.

(The solutions of $v = 3t^2 - 12t + 5 = 0$ are approximately 7.79 and 0.21, so P does not turn round in the interval.)

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Exercise B, Question 11

Question:

A particle P moves in a straight line so that, at time t seconds, its velocity v m s^{-1} is given by

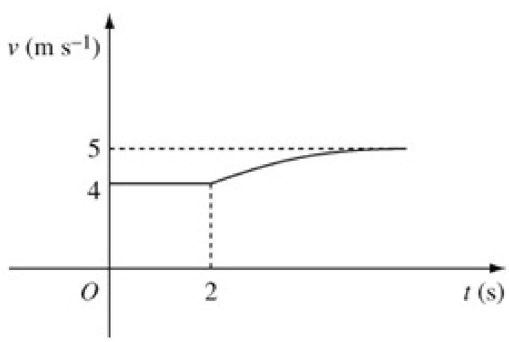
$$v = \begin{cases} 4, & 0 \le t \le 2 \\ 5 - \dfrac{4}{t^2}, & t > 2. \end{cases}$$

a Sketch a velocity-time graph to illustrate the motion of P.

b Find the distance moved by P in the interval $0 \le t \le 5$.

Solution:

a



b In the first two seconds P moves $2 \times 4 = 8$ m

$s = \int v \, dt = \int (5 - 4t^{-2}) \, dt$

$= 5t - \frac{4t^{-1}}{-1} + C = 5t + \frac{4}{t} + C$

When $t = 2, s = 8$

$8 = 5 \times 2 + \frac{4}{2} + C = 12 + C \Rightarrow C = -4$

$s = 5t + \frac{4}{t} - 4$

When $t = 5$,

$s = 5 \times 5 + \frac{4}{5} - 4 = 21.8$

In the interval $0 \le t \le 5$, P moves 21.8 m.

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Exercise B, Question 12

Question:

A particle P moves in a straight line so that, at time t seconds, its acceleration, a m s^{-2}, is given by

$$a = \begin{cases} 6t - t^2, & 0 \le t \le 2 \\ 8 - t, & t > 2. \end{cases}$$

When $t = 0$ the particle is at rest at a fixed point O on the line. Find

a the speed of P when $t = 2$,

b the speed of P when $t = 4$,

c the distance from O to P when $t = 4$.

Solution:

a For $0 \le t \le 2$

$v = \int a \, dt = \int (6t - t^2) dt$

$= 3t^2 - \frac{1}{3}t^3 + c$, where c is a constant of integration.

When $t = 0, v = 0$

$0 = 0 - 0 + c \Rightarrow c = 0$

$v = 3t^2 - \frac{1}{3}t^3$

When $t = 2$,

$v = 3 \times 2^2 - \frac{2^3}{3} = \frac{28}{3}$

The speed of P when $t = 2$ is $\frac{28}{3}$ m s^{-1}.

b For $t > 2$,

$v = \int a dt = \int (8 - t) dt$

$= 8t - \frac{1}{2}t^2 + k$, where k is a constant of integration.

From **a**, when $t = 2, v = \frac{28}{3}$

$\frac{28}{3} = 16 - \frac{4}{2} + k \Rightarrow k = -\frac{14}{3}$

$v = 8t - \frac{1}{2}t^2 - \frac{14}{3}$

When $t = 4$,

$v = 32 - 8 - \frac{14}{3} = \frac{58}{3}$

The speed of P when $t = 4$ is $\frac{58}{3}$ m s^{-1}.

c For $0 \le t \le 2$,

$x = \int v dt = \int \left(3t^2 - \frac{1}{3}t^3\right) dt = t^3 - \frac{1}{12}t^4 + l$, where l is a constant of integration.

When $t = 0, x = 0$

$0 = 0 - 0 + l \Rightarrow l = 0$

When $t = 2$,

$x = 2^3 - \frac{2^4}{12} = \frac{20}{3}$ (1)

For $t > 2$,

$x = \int v dt = \int \left(8t - \frac{1}{2}t^2 - \frac{14}{3}\right) dt = 4t^2 - \frac{1}{6}t^3 - \frac{14}{3}t + m$,
where m is a constant of integration.

From (1) above

When $t = 2, x = \frac{20}{3}$

$\frac{20}{3} = 16 - \frac{8}{6} - \frac{28}{3} + m \Rightarrow m = \frac{4}{3}$

$x = 4t^2 - \frac{1}{6}t^3 - \frac{14}{3}t + \frac{4}{3}$

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

At time t seconds, a particle P has position vector $\mathbf{r}$ m with respect to a fixed origin O, where

$$\mathbf{r} = (3t - 4)\mathbf{i} + (t^3 - 4t)\mathbf{j}.$$

Find

a the velocity of P when $t = 3$,

b the acceleration of P when $t = 3$.

Solution:

a $\mathbf{v} = \dot{\mathbf{r}} = 3\mathbf{i} + (3t^2 - 4)\mathbf{j}$

When $t = 3$,

$\mathbf{v} = 3\mathbf{i} + 23\mathbf{j}$

The velocity of P when $t = 3$ is $(3\mathbf{i} + 23\mathbf{j})$ m s^{-1}.

b $\mathbf{a} = \dot{\mathbf{v}} = 6t\mathbf{j}$

When $t = 3$,

$\mathbf{a} = 18\mathbf{j}$

The acceleration of P when $t = 3$ is $18\mathbf{j}$ m s^{-2}.

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Exercise C, Question 2

Question:

A particle P is moving in a plane with velocity $\mathbf{v}$ m s^{-1} at time t seconds where

$$\mathbf{v} = t^2\mathbf{i} + (2t - 3)\mathbf{j}.$$

When $t = 0$, P has position vector $(3\mathbf{i} + 4\mathbf{j})$ m with respect to a fixed origin O. Find

a the acceleration of P at time t seconds,

b the position vector of P when $t = 1$.

Solution:

a $\mathbf{a} = \dot{\mathbf{v}} = 2t\mathbf{i} + 2\mathbf{j}$

The acceleration of P at time t seconds $(2t\mathbf{i} + 2\mathbf{j})$ m s^{-2}.

b

$$\mathbf{r} = \int \mathbf{v}\, dt = \int (t^2\mathbf{i} + (2t - 3)\mathbf{j})\, dt$$

$$= \frac{t^3}{3}\mathbf{i} + (t^2 - 3t)\mathbf{j} + \mathbf{C}$$

When $t = 0, \mathbf{r} = 3\mathbf{i} + 4\mathbf{j}$

$3\mathbf{i} + 4\mathbf{j} = 0\mathbf{i} + 0\mathbf{j} + \mathbf{C} \Rightarrow \mathbf{C} = 3\mathbf{i} + 4\mathbf{j}$

Hence

$$\mathbf{r} = \left(\frac{t^3}{3} + 3\right)\mathbf{i} + (t^2 - 3t + 4)\mathbf{j}$$

When $t = 1$

$$\mathbf{r} = 3\tfrac{1}{3}\mathbf{i} + 2\mathbf{j}$$

The position vector of P when $t = 1$ is $\left(3\tfrac{1}{3}\mathbf{i} + 2\mathbf{j}\right)$m.

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Exercise C, Question 3

Question:

A particle P starts from rest at a fixed origin O. The acceleration of P at time t seconds (where $t \geq 0$) is $(6t^2\mathbf{i} + (8 - 4t^3)\mathbf{j})$ m s^{-2}. Find

a the velocity of P when $t = 2$,

b the position vector of P when $t = 4$.

Solution:

a

$$\mathbf{v} = \int \mathbf{a}\,dt = \int (6t^2\mathbf{i} + (8 - 4t^3)\mathbf{j})\,dt$$
$$= 2t^3\mathbf{i} + (8t - t^4)\mathbf{j} + \mathbf{C}$$

When $t = 0, \mathbf{v} = 0\mathbf{i} + 0\mathbf{j}$

$$0\mathbf{i} + 0\mathbf{j} = 0\mathbf{i} + 0\mathbf{j} + \mathbf{C} \Rightarrow \mathbf{C} = 0\mathbf{i} + 0\mathbf{j}$$

Hence

$$\mathbf{v} = 2t^3\mathbf{i} + (8t - t^4)\mathbf{j}$$

When $t = 2$

$$\mathbf{v} = 16\mathbf{i} + (8 \times 2 - 2^4)\mathbf{j} = 16\mathbf{i}$$

The velocity of P when $t = 2$ is $16\mathbf{i}$ m s^{-1}.

b

$$\mathbf{r} = \int \mathbf{v}\,dt = \int (2t^3\mathbf{i} + (8t - t^4)\mathbf{j})dt$$
$$= \frac{1}{2}t^4\mathbf{i} + \left(4t^2 - \frac{1}{5}t^5\right)\mathbf{j} + \mathbf{D}$$

When $t = 0, \mathbf{r} = 0\mathbf{i} + 0\mathbf{j}$

$$0\mathbf{i} + 0\mathbf{j} = 0\mathbf{i} + 0\mathbf{j} + \mathbf{D} \Rightarrow \mathbf{D} = 0\mathbf{i} + 0\mathbf{j}$$

Hence

$$\mathbf{r} = \frac{t^4}{2}\mathbf{i} + \left(4t^2 - \frac{t^5}{5}\right)\mathbf{j}$$

When $t = 4$

$$\mathbf{r} = \frac{4^4}{2}\mathbf{i} + \left(4 \times 4^2 - \frac{4^5}{5}\right)\mathbf{j} = 128\mathbf{i} - 104.8\mathbf{j}$$

The position vector of P when $t = 4$ is $(128\mathbf{i} - 104.8\mathbf{j})$m.

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 4

Question:

At time t seconds, a particle P has position vector $\mathbf{r}$ m with respect to a fixed origin O, where

$$\mathbf{r} = 4t^2\mathbf{i} + (24t - 3t^2)\mathbf{j}.$$

a Find the speed of P when $t = 2$.

b Show that the acceleration of P is a constant and find the magnitude of this acceleration.

Solution:

a $\mathbf{v} = \dot{\mathbf{r}} = 8t\mathbf{i} + (24 - 6t)\mathbf{j}$

When $t = 2$

$\mathbf{v} = 16\mathbf{i} + 12\mathbf{j}$

$|\mathbf{v}|^2 = 16^2 + 12^2 = 400 \Rightarrow |\mathbf{v}| = \sqrt{400} = 20$

The speed of P when $t = 2$ is 20 m s^{-1}.

b $\mathbf{a} = \dot{\mathbf{v}} = 8\mathbf{i} - 6\mathbf{j}$

As there is no t in this expression, the acceleration is a constant.

$|\mathbf{a}|^2 = 8^2 + (-6)^2 = 100 \Rightarrow |\mathbf{a}| = \sqrt{100} = 10$

The magnitude of the acceleration is 10 m s^{-2}.

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Exercise C, Question 5

Question:

A particle P is initially at a fixed origin O. At time $t = 0$, P is projected from O and moves so that, at time t seconds after projection, its position vector $\mathbf{r}$ m relative to O is given by

$$\mathbf{r} = (t^3 - 12t)\mathbf{i} + (4t^2 - 6t)\mathbf{j},\ t \geq 0.$$

Find

a the speed of projection of P,

b the value of t at the instant when P is moving parallel to $\mathbf{j}$,

c the position vector of P at the instant when P is moving parallel to $\mathbf{j}$.

Solution:

a $\mathbf{v} = \dot{\mathbf{r}} = (3t^2 - 12)\mathbf{i} + (8t - 6)\mathbf{j}$

When $t = 0$,

$\mathbf{v} = -12\mathbf{i} - 6\mathbf{j}$

$|\mathbf{v}|^2 = (-12)^2 + (-6)^2 = 180 \Rightarrow |\mathbf{v}| = \sqrt{180} = 6\sqrt{5}$

The speed of projection is $6\sqrt{5}$ m s^{-1}.

b When P is moving parallel to $\mathbf{j}$ the velocity has no $\mathbf{i}$ component.

$3t^2 - 12 = 0 \Rightarrow t^2 = 4 \Rightarrow t = 2 \quad (t \geq 0)$

c When $t = 2$

$\mathbf{r} = (2^3 - 12 \times 2)\mathbf{i} + (4 \times 2^2 - 6 \times 2)\mathbf{j} = -16\mathbf{i} + 4\mathbf{j}$

The position vector of P at the instant when P is moving parallel to $\mathbf{j}$ is $(-16\mathbf{i} + 4\mathbf{j})$m .

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Exercise C, Question 6

Question:

At time t seconds, the force $\mathbf{F}$ newtons acting on a particle P, of mass 0.5 kg, is given by

$$\mathbf{F} = 3t\mathbf{i} + (4t - 5)\mathbf{j}.$$

When $t = 1$, the velocity of P is $12\mathbf{i}$ m s^{-1}. Find

a the velocity of P after t seconds,

b the angle the direction of motion of P makes with $\mathbf{i}$ when $t = 5$, giving your answer to the nearest degree.

Solution:

a

$$\mathbf{F} = m\mathbf{a}$$

$$3t\mathbf{i} + (4t - 5)\mathbf{j} = 0.5\mathbf{a}$$

$$\mathbf{a} = 6t\mathbf{i} + (8t - 10)\mathbf{j}$$

$$\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t = \int (6t\mathbf{i} + (8t - 10)\mathbf{j})\,\mathrm{d}t$$

$$= 3t^2\mathbf{i} + (4t^2 - 10t)\mathbf{j} + \mathbf{C}$$

When $t = 1, \mathbf{v} = 12\mathbf{i}$

$$12\mathbf{i} = 3\mathbf{i} - 6\mathbf{j} + \mathbf{C} \Rightarrow \mathbf{C} = 9\mathbf{i} + 6\mathbf{j}$$

Hence

$$\mathbf{v} = (3t^2 + 9)\mathbf{i} + (4t^2 - 10t + 6)\mathbf{j}$$

When $t = 5$

$$\mathbf{v} = (3 \times 5^2 + 9)\mathbf{i} + (4 \times 5^2 - 10 \times 5 + 6)\mathbf{j} = 84\mathbf{i} + 56\mathbf{j}$$

b The angle $\mathbf{v}$ makes with $\mathbf{i}$ is given by

$$\tan\theta = \frac{56}{84} \Rightarrow \theta \approx 34^\circ$$

The angle the direction of motion of P makes with $\mathbf{i}$ when $t = 5$ is 34° (nearest degree).

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Exercise C, Question 7

Question:

A particle P is moving in a plane with velocity $\mathbf{v}$ m s^{-1} at time t seconds where

$$\mathbf{v} = (3t^2 + 2)\mathbf{i} + (6t - 4)\mathbf{j}.$$

When $t = 2$, P has position vector $9\mathbf{j}$ m with respect to a fixed origin O. Find

a the distance of P from O when $t = 0$,

b the acceleration of P at the instant when it is moving parallel to the vector $\mathbf{i}$.

Solution:

a

$$\mathbf{r} = \int \mathbf{v}\, dt = \int ((3t^2 + 2)\mathbf{i} + (6t - 4)\mathbf{j})dt$$
$$= (t^3 + 2t)\mathbf{i} + (3t^2 - 4t)\mathbf{j} + \mathbf{A}$$

When $t = 2, \mathbf{v} = 9\mathbf{j}$

$$9\mathbf{j} = 12\mathbf{i} + 4\mathbf{j} + \mathbf{A} \Rightarrow \mathbf{A} = -12\mathbf{i} + 5\mathbf{j}$$

Hence

$$\mathbf{r} = (t^3 + 2t - 12)\mathbf{i} + (3t^2 - 4t + 5)\mathbf{j}$$

When $t = 0$,

$$\mathbf{r} \quad = -12\mathbf{i} + 5\mathbf{j}$$

$$|\mathbf{r}|^2 = (-12)^2 + 5^2 = 169 \Rightarrow |\mathbf{r}| = \sqrt{169} = 13$$

The distance of P from O when $t = 0$ is 13 m.

b When P is moving parallel to $\mathbf{i}$, $\mathbf{v}$ has no $\mathbf{j}$ component.

$$6t - 4 = 0 \Rightarrow t = \frac{2}{3}$$
$$\mathbf{a} = \dot{\mathbf{v}} = 6t\mathbf{i} + 6\mathbf{j}$$

When $t = \frac{2}{3}$,

$$\mathbf{a} = 4\mathbf{i} + 6\mathbf{j}$$

The acceleration of P at the instant when it is moving parallel to the vector $\mathbf{i}$ is $(4\mathbf{i} + 6\mathbf{j})$ m s^{-2}.

Solutionbank M2
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Exercise C, Question 8

Question:

At time t seconds, the particle P is moving in a plane with velocity $\mathbf{v}$ m s^{-1} and acceleration $\mathbf{a}$ m s^{-2}, where

$$\mathbf{a} = (2t-4)\mathbf{i} + 6\mathbf{j}.$$

Given that P is instantaneously at rest when $t = 4$, find

a $\mathbf{v}$ in terms of t,

b the speed of P when $t = 5$.

Solution:

a $\mathbf{v} = \int \mathbf{a}\,dt = \int((2t-4)\mathbf{i} + 6\mathbf{j})\,dt = (t^2 - 4t)\mathbf{i} + 6t\mathbf{j} + \mathbf{C}$

When $t = 4$, $\mathbf{v} = 0\mathbf{i} + 0\mathbf{j}$

$0\mathbf{i} + 0\mathbf{j} = (4^2 - 4\times 4)\mathbf{i} + 6\times 4\mathbf{j} + \mathbf{C} = 24\mathbf{j} + \mathbf{C} \Rightarrow \mathbf{C} = -24\mathbf{j}$

Hence

$\mathbf{v} = (t^2 - 4t)\mathbf{i} + (6t - 24)\mathbf{j}$

b When $t = 5$

$\mathbf{v} = 5\mathbf{i} + 6\mathbf{j}$

$|\mathbf{v}|^2 = 5^2 + 6^2 = 61 \Rightarrow |\mathbf{v}| = \sqrt{61} \approx 7.81$

The speed of P when $t = 5$ is 7.81 m s^{-1} (3 s.f.).

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Exercise C, Question 9

Question:

A particle P is moving in a plane. At time t seconds, the position vector of P, $\mathbf{r}$ m, is given by

$\mathbf{r} = (3t^2 - 6t + 4)\mathbf{i} + (t^3 + kt^2)\mathbf{j}$, where k is a constant.

When $t = 3$, the speed of P is $12\sqrt{5}$ m s^{-1}.

a Find the two possible values of k.

b For both of these values of k, find the magnitude of the acceleration of P when $t = 1.5$.

Solution:

a $\mathbf{v} = \dot{\mathbf{r}} = (6t - 6)\mathbf{i} + (3t^2 + 2kt)\mathbf{j}$

When $t = 3$

$\mathbf{v} = 12\mathbf{i} + (27 + 6k)\mathbf{j}$

$|\mathbf{v}|^2 = 12^2 + (27 + 6k)^2 = (12\sqrt{5})^2$

$144 + 729 + 324k + 36k^2 = 720$

$36k^2 + 324k + 153 = 0$

$(\div 9)$

$4k^2 + 36k + 17 = (2k + 1)(2k + 17) = 0$

$k = -0.5, -8.5$

b If $k = -0.5$

$\mathbf{v} = (6t - 6)\mathbf{i} + (3t^2 - t)\mathbf{j}$

$\mathbf{a} = \dot{\mathbf{v}} = 6\mathbf{i} + (6t - 1)\mathbf{j}$

When $t = 1.5$

$\mathbf{a} = 6\mathbf{i} + 8\mathbf{j}$

$|\mathbf{a}|^2 = 6^2 + 8^2 = 100 \Rightarrow |\mathbf{a}| = 10$

If $k = -8.5$

$\mathbf{v} = (6t - 6)\mathbf{i} + (3t^2 - 17t)\mathbf{j}$

$\mathbf{a} = \dot{\mathbf{v}} = 6\mathbf{i} + (6t - 17)\mathbf{j}$

When $t = 1.5$

$\mathbf{a} = 6\mathbf{i} - 8\mathbf{j}$

$|\mathbf{a}|^2 = 6^2 + (-8)^2 = 100 \Rightarrow |\mathbf{a}| = 10$

For both of the values of k the magnitude of the acceleration of P when $t = 1.5$ is 10 m s^{-2}.

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Exercise C, Question 10

Question:

At time t seconds (where $t \geq 0$), the particle P is moving in a plane with acceleration $\mathbf{a}$ m s^{-2}, where

$$\mathbf{a} = (5t - 3)\mathbf{i} + (8 - t)\mathbf{j}$$

When $t = 0$, the velocity of P is $(2\mathbf{i} - 5\mathbf{j})$ m s^{-1}. Find

a the velocity of P after t seconds,

b the value of t for which P is moving parallel to $\mathbf{i} - \mathbf{j}$,

c the speed of P when it is moving parallel to $\mathbf{i} - \mathbf{j}$.

Solution:

a

$$\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t = \int ((5t-3)\mathbf{i} + (8-t)\mathbf{j})\,\mathrm{d}t$$

$$= \left(\frac{5}{2}t^2 - 3t\right)\mathbf{i} + \left(8t - \frac{1}{2}t^2\right)\mathbf{j} + \mathbf{C}$$

When $t = 0, \mathbf{v} = 2\mathbf{i} - 5\mathbf{j}$

$2\mathbf{i} - 5\mathbf{j} = 0\mathbf{i} + 0\mathbf{j} + \mathbf{C} \Rightarrow \mathbf{C} = 2\mathbf{i} - 5\mathbf{j}$

Hence

$$\mathbf{v} = \left(\frac{5}{2}t^2 - 3t + 2\right)\mathbf{i} + \left(8t - \frac{1}{2}t^2 - 5\right)\mathbf{j}$$

The velocity of P after t seconds is $\left(\left(\frac{5}{2}t^2 - 3t + 2\right)\mathbf{i} + \left(8t - \frac{1}{2}t^2 - 5\right)\mathbf{j}\right)$ m s^{-1}.

b The gradients of $\mathbf{v}$ and $\mathbf{i} - \mathbf{j}$ are equal

$$\frac{8t - \frac{1}{2}t^2 - 5}{\frac{5}{2}t^2 - 3t + 2} = 1$$

$$8t - \frac{1}{2}t^2 - 5 = -\frac{5}{2}t^2 + 3t - 2$$

$$2t^2 + 5t - 3 = (2t-1)(t+3) = 0$$

$$t = \frac{1}{2}, -3$$

As $t \geq 0, t = \frac{1}{2}$

c When $t = \frac{1}{2}$

$$\mathbf{v} = \left(\frac{5}{8} - \frac{3}{2} + 2\right)\mathbf{i} + \left(4 - \frac{1}{8} - 5\right)\mathbf{j} = \frac{9}{8}\mathbf{i} - \frac{9}{8}\mathbf{j}$$

$$|\mathbf{v}|^2 = \left(\frac{9}{8}\right)^2 + \left(-\frac{9}{8}\right)^2 = 2 \times \left(\frac{9}{8}\right)^2 \Rightarrow |\mathbf{v}| = \frac{9\sqrt{2}}{8}$$

The speed of P when it is moving parallel to $\mathbf{i} - \mathbf{j}$ is $\frac{9\sqrt{2}}{8}$ m s^{-1}.

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 11

Question:

At time t seconds (where $t \geq 0$), a particle P is moving in a plane with acceleration $(2\mathbf{i} - 2t\mathbf{j})$ m s^{-2}. When $t = 0$, the velocity of P is $2\mathbf{j}$ m s^{-1} and the position vector of P is $6\mathbf{i}$ m with respect to a fixed origin P.

a Find the position vector of P at time t seconds.

At time t seconds (where $t \geq 0$), a second particle Q is moving in the plane with velocity $((3t^2 - 4)\mathbf{i} - 2t\mathbf{j})$ m s^{-1}. The particles collide when $t = 3$.

b Find the position vector of Q at time $t = 0$.

Solution:

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Exercise C, Question 12

Question:

A particle P of mass 0.2 kg is at rest at a fixed origin O. At time t seconds, where $0 \le t \le 3$, a force $(2t\mathbf{i} + 3\mathbf{j})$ N is applied to P.

a Find the position vector of P when $t = 3$.

When $t = 3$, the force acting on P changes to $(6\mathbf{i} + (12 - t^2)\mathbf{j})$ N, where $t \ge 3$.

b Find the velocity of P when $t = 6$.

Solution:

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Exercise D, Question 1

Question:

Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.

A particle P is projected from a point O on a horizontal plane with speed 42 m s^{-1} and with angle of elevation $45°$. After projection, the particle moves freely under gravity until it strikes the plane. Find

a the greatest height above the plane reached by P,

b the time of flight of P.

Solution:

a Resolving the initial velocity vertically

$\text{R}(\uparrow)$ $u_y = 42 \sin 45° = 21\sqrt{2}$

$\text{R}(\uparrow)$ $u = 21\sqrt{2}, v = 0, a = -9.8, s = ?$

$$v^2 = u^2 + 2as$$

$$0^2 = (21\sqrt{2})^2 - 2 \times 9.8 \times s$$

$$s = \frac{(21\sqrt{2})^2}{2 \times 9.8} = \frac{882}{19.6} = 45$$

The greatest height above the plane reached by P is 45 m.

b

$\text{R}(\uparrow)$ $s = 0, u = 21\sqrt{2}, a = -9.8, t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$0 = 21\sqrt{2}t - 4.9t^2$$

$t \neq 0$

$$t = \frac{21\sqrt{2}}{4.9} = 6.0609\ldots$$

The time of flight of P is 6.1 s (2 s.f.).

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Exercise D, Question 2

Question:

A stone is thrown horizontally with speed 21 m s^{-1} from a point P on the edge of a cliff h metres above sea level. The stone lands in the sea at a point Q, where the horizontal distance of Q from the cliff is 56 m.

Calculate the value of h.

Solution:

Resolving the initial velocity horizontally and vertically

$\text{R}(\rightarrow) \quad u_x = 21$

$\text{R}(\downarrow) \quad u_y = 0$

$\text{R}(\rightarrow) \quad \text{distance} = \text{speed} \times \text{time}$

$$56 = 21 \times t \Rightarrow t = \frac{56}{21} = \frac{8}{3}$$

$\text{R}(\downarrow) \quad s = h, u = 0, a = 9.8, t = \frac{8}{3}$

$$s = ut + \frac{1}{2}at^2$$

$$h = 0 + 4.9 \times \left(\frac{8}{3}\right)^2 = 34.844\ldots$$

$$h = 35 \text{ (2 s.f.)}$$

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Exercise D, Question 3

Question:

A particle P moves in a horizontal straight line. At time t seconds (where $t \geq 0$) the velocity v m s^{-1} of P is given by $v = 15 - 3t$. Find

a the value of t when P is instantaneously at rest,

b the distance travelled by P between the time when $t = 0$ and the time when P is instantaneously at rest.

Solution:

a $v = 15 - 3t$

When P is at rest, $v = 0$

$0 = 15 - 3t \Rightarrow t = 5$

b

$s = \int v \, dt = \int (15 - 3t) \, dt$

$= 15t - \frac{3}{2}t^2 + c$

Let $s = 0$, when $t = 0$

$0 = 0 - 0 + c \Rightarrow c = 0$

$s = 15t - \frac{3}{2}t^2$

When $t = 5$

$s = 15 \times 5 - \frac{3}{2}5^2 = 37.5$

The distance travelled by P between the time when $t = 0$ and the time when P is instantaneously at rest is 37.5 m.

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Exercise D, Question 4

Question:

A particle P moves along the x-axis so that, at time t seconds, the displacement of P from O is x metres and the velocity of P is v m s^{-1}, where

$$v = 6t + \frac{1}{2}t^3.$$

a Find the acceleration of P when $t = 4$.

b Given also that $x = -5$ when $t = 0$, find the distance OP when $t = 4$.

Solution:

a $a = \frac{dv}{dt} = 6 + \frac{3}{2}t^2$

When $t = 4$

$$a = 6 + \frac{3}{2}4^2 = 30$$

The acceleration of P when $t = 4$ is 30 m s^{-2}.

b

$$x = \int v \, dt = \int \left(6t + \frac{1}{2}t^3\right) dt$$

$$= 3t^2 + \frac{1}{8}t^4 + c$$

When $t = 0, x = -5$

$$-5 = 0 + 0 + c \Rightarrow c = -5$$

$$x = 3t^2 + \frac{1}{8}t^4 - 5$$

When $t = 4$

$$x = 3 \times 4^2 + \frac{4^4}{8} - 5 = 75$$

$$OP = 75 \text{ m}$$

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Exercise D, Question 5

Question:

At time t seconds, a particle P has position vector $\mathbf{r}$ m with respect to a fixed origin O, where

$$\mathbf{r} = (3t^2 - 4)\mathbf{i} + (8 - 4t^2)\mathbf{j}.$$

a Show that the acceleration of P is a constant.

b Find the magnitude of the acceleration of P and the size of the angle which the acceleration makes with $\mathbf{j}$.

Solution:

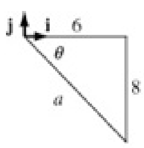
a

$\mathbf{v} = \dot{\mathbf{r}} = 6t\mathbf{i} - 8t\mathbf{j}$

$\mathbf{a} = \dot{\mathbf{v}} = 6\mathbf{i} - 8\mathbf{j}$

Acceleration does not depend on t, hence the acceleration is a constant.

b



$|\mathbf{a}| = 6^2 + (-8)^2 = 100 \Rightarrow |\mathbf{a}| = 10$

The magnitude of the acceleration is 10 m s^{-2}.

$\tan\theta = \frac{8}{6} \Rightarrow \theta \approx 53.1^\circ$

The angle the acceleration makes with $\mathbf{j}$ is $90^\circ + 53.1^\circ = 143.1^\circ$ (nearest 0.1°).

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Exercise D, Question 6

Question:

At time $t = 0$ a particle P is at rest at a point with position vector $(4\mathbf{i} - 6\mathbf{j})$ m with respect to a fixed origin O. The acceleration of P at time t seconds (where $t \geq 0$) is $((4t - 3)\mathbf{i} - 6t^2\mathbf{j})$ m s^{-2}. Find

a the velocity of P when $t = \frac{1}{2}$,

b the position vector of P when $t = 6$.

Solution:

a

$$\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t = \int ((4t - 3)\mathbf{i} - 6t^2\mathbf{j})\,\mathrm{d}t$$
$$= (2t^2 - 3t)\mathbf{i} - 2t^3\mathbf{j} + \mathbf{A}$$

When $t = 0, \mathbf{v} = \mathbf{0}$

$$\mathbf{0} = 0\mathbf{i} + 0\mathbf{j} + \mathbf{A} \Rightarrow \mathbf{A} = \mathbf{0}$$
$$\mathbf{v} = (2t^2 - 3t)\mathbf{i} - 2t^3\mathbf{j}$$

When $t = \frac{1}{2}$

$$\mathbf{v} = \left(2\left(\frac{1}{2}\right)^2 - 3 \times \frac{1}{2}\right)\mathbf{i} - 2\left(\frac{1}{2}\right)^3\mathbf{j} = -\mathbf{i} - \frac{1}{4}\mathbf{j}$$

The velocity of P when $t = \frac{1}{2}$ is $\left(-\mathbf{i} - \frac{1}{4}\mathbf{j}\right)$ m s^{-1}.

b

$$\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t = \int \left(\left(2t^2 - 3t\right)\mathbf{i} - 2t^3\mathbf{j}\right)\mathrm{d}t$$
$$= \left(\frac{2}{3}t^3 - \frac{3}{2}t^2\right)\mathbf{i} - \frac{1}{2}t^4\mathbf{j} + \mathbf{B}$$

When $t = 0, \mathbf{r} = 4\mathbf{i} - 6\mathbf{j}$

$$4\mathbf{i} - 6\mathbf{j} = 0\mathbf{i} + 0\mathbf{j} + \mathbf{B} \Rightarrow \mathbf{B} = 4\mathbf{i} - 6\mathbf{j}$$
$$\mathbf{r} = \left(\frac{2}{3}t^3 - \frac{3}{2}t^2 + 4\right)\mathbf{i} - \left(\frac{1}{2}t^4 + 6\right)\mathbf{j}$$

When $t = 6$

$$\mathbf{r} = (144 - 54 + 4)\mathbf{i} - (648 + 6)\mathbf{j} = 94\mathbf{i} - 654\mathbf{j}$$

The position vector of P when $t = 6$ is $(94\mathbf{i} - 654\mathbf{j})$ m.

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Exercise D, Question 7

Question:

A ball is thrown from a window above a horizontal lawn. The velocity of projection is 15 m s^{-1} and the angle of elevation is α, where $\tan\alpha = \frac{4}{3}$. The ball takes 4 s to reach the lawn. Find

a the horizontal distance between the point of projection and the point where the ball hits the lawn,

b the vertical height above the lawn from which the ball was thrown.

Solution:

a $\tan\alpha = \frac{4}{3} \Rightarrow \sin\alpha = \frac{4}{5}, \cos\alpha = \frac{3}{5}$

Resolving the initial velocity horizontally and vertically

R($\rightarrow$) $u_x = 15\cos\alpha = 15\times\frac{3}{5} = 9$

R($\uparrow$) $u_y = 15\sin\alpha = 15\times\frac{4}{5} = 12$

R($\rightarrow$) distance = speed $\times$ time

$= 9\times 4 = 36$

The horizontal distance between the point of projection and the point where the ball hits the lawn is 36 m.

b Let the vertical height above the lawn from which the ball was thrown be h m

R($\uparrow$) $s = -h, u = 12, a = -9.8, t = 4$

$s = ut + \frac{1}{2}at^2$

$-h = 12\times 4 - 4.9\times 4^2 = -30.4 \Rightarrow h = 30.4$

The vertical height above the lawn from which the ball was thrown is 30 m (2 s.f.).

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Exercise D, Question 8

Question:

A projectile is fired with velocity 40 m s^{-1} at an angle of elevation of $30°$ from a point A on horizontal ground. The projectile moves freely under gravity until it reaches the ground at the point B. Find

a the distance AB,

b the speed of the projectile at the instants when it is 15 m above the plane.

Solution:

a Resolving the initial velocity horizontally and vertically

$$\text{R}(\rightarrow) \quad u_x = 40\cos 30° = 20\sqrt{3}$$
$$\text{R}(\uparrow) \quad u_y = 20\sin 30° = 10$$
$$\text{R}(\uparrow) \quad s = 0, u = 20, a = -9.8, t = ?$$
$$s = ut + \frac{1}{2}at^2$$
$$0 = 20t - 4.9t^2 = t(20 - 4.9t)$$

$$t \neq 0$$
$$t = \frac{20}{4.9}$$
$$\text{R}(\rightarrow) \quad \text{distance} = \text{speed} \times \text{time}$$
$$= 20\sqrt{3} \times \frac{20}{4.9} = 141.39\ldots$$
$$AB = 140 \text{ (2 s.f.)}$$

b

$$\text{R}(\uparrow) \quad u = 20, a = -9.8, s = 15, v = v_y$$
$$v^2 = u^2 + 2as$$
$$v_y^2 = 20^2 - 2 \times 9.8 \times 15 = 106$$
$$V^2 = u_x^2 + v_y^2 = (20\sqrt{3})^2 + 106 = 1306$$
$$V = \sqrt{1306} = 36.138\ldots$$

The speed of the projectile at the instants when it is 15 m above the plane is 36 m s^{-1} (2 s.f.).

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Exercise D, Question 9

Question:

At time t seconds, a particle P has position vector $\mathbf{r}$ m with respect to a fixed origin O, where

$$\mathbf{r} = 2\cos 3t\mathbf{i} - 2\sin 3t\mathbf{j}.$$

a Find the velocity of P when $t = \frac{\pi}{6}$.

b Show that the magnitude of the acceleration of P is constant.

Solution:

a $\mathbf{v} = \dot{\mathbf{r}} = -6\sin 3t\mathbf{i} - 6\cos 3t\mathbf{j}$

When $t = \frac{\pi}{6}$

$$\mathbf{v} = \dot{\mathbf{r}} = -6\sin\frac{\pi}{2}\mathbf{i} - 6\cos\frac{\pi}{2}\mathbf{j} = -6\mathbf{i} - 0\mathbf{j}$$

The velocity of P when $t = \frac{\pi}{6}$ is $-6\mathbf{i}$ m s^{-1}.

b

$$\mathbf{a} = \dot{\mathbf{v}} = -18\cos 3t\mathbf{i} + 18\sin 3t\mathbf{j}$$
$$|\mathbf{a}|^2 = (-18\cos 3t)^2 + (18\sin 3t)^2$$
$$= 18^2(\cos^2 3t + \sin^2 3t) = 18^2$$
$$|\mathbf{a}| = 18$$

The magnitude of the acceleration is 18 m s^{-2}, a constant.

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Exercise D, Question 10

Question:

A particle P of mass 0.2 kg is moving in a straight line under the action of a single variable force $\mathbf{F}$ newtons. At time t seconds the displacement, s metres, of P from a fixed point A is given by $s = 3t + 4t^2 - \frac{1}{2}t^3$.

Find the magnitude of $\mathbf{F}$ when $t = 4$.

Solution:

$v = \frac{ds}{dt} = 3 + 4t - \frac{3}{2}t^2$

$a = \frac{dv}{dt} = 8 - 3t$

When $t = 4$

$a = 8 - 3 \times 4 = -4$

$\mathbf{F} = m\mathbf{a}$

$= 0.2 \times (-4) = -0.8$

The magnitude of $\mathbf{F}$ when $t = 4$ is 0.8 N.

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Exercise D, Question 11

Question:

At time t seconds (where $t \geq 0$) the particle P is moving in a plane with acceleration $\mathbf{a}$ m s^{-2}, where

$$\mathbf{a} = (8t^3 - 6t)\mathbf{i} + (8t - 3)\mathbf{j}.$$

When $t = 2$, the velocity of P is $(16\mathbf{i} + 3\mathbf{j})$ m s^{-1}. Find

a the velocity of P after t seconds,

b the value of t when P is moving parallel to $\mathbf{i}$.

Solution:

a

$$\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t = \int \left[(8t^3 - 6t)\mathbf{i} + (8t - 3)\mathbf{j} \right] \mathrm{d}t$$

$$= \left(2t^4 - 3t^2\right)\mathbf{i} + \left(4t^2 - 3t\right)\mathbf{j} + \mathbf{C}$$

When $t = 2, \mathbf{v} = 16\mathbf{i} + 3\mathbf{j}$

$$16\mathbf{i} + 3\mathbf{j} = 20\mathbf{i} + 10\mathbf{j} + \mathbf{C} \Rightarrow \mathbf{C} = -4\mathbf{i} - 7\mathbf{j}$$

$$\mathbf{v} = (2t^4 - 3t^2 - 4)\mathbf{i} + (4t^2 - 3t - 7)\mathbf{j}$$

The velocity of P after t seconds is $\left[\left(2t^4 - 3t^2 - 4\right)\mathbf{i} + \left(4t^2 - 3t - 7\right)\mathbf{j}\right]$ m s^{-1}.

b When P is moving parallel to $\mathbf{i}$, the $\mathbf{j}$ component of the velocity is zero.

$$4t^2 - 3t - 7 = 0$$

$$(t + 1)(4t - 7) = 0$$

$$t \geq 0$$

$$t = \frac{7}{4}$$

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Exercise D, Question 12

Question:

A particle of mass 0.5 kg is acted upon by a variable force $\mathbf{F}$. At time t seconds, the velocity $\mathbf{v}$ m s^{-1} is given by
$\mathbf{v} = (4ct - 6)\mathbf{i} + (7 - c)t^2\mathbf{j}$, where c is a constant.

a Show that $\mathbf{F} = [2c\mathbf{i} + (7 - c)t\mathbf{j}]$ N.

b Given that when $t = 5$ the magnitude of $\mathbf{F}$ is 17 N, find the possible values of c.

Solution:

a

$\mathbf{a} = \dot{\mathbf{v}} = 4c\mathbf{i} + 2(7 - c)t\mathbf{j}$

$\mathbf{F} = m\mathbf{a}$

$= 0.5\left[4c\mathbf{i} + 2(7 - c)t\mathbf{j}\right] = 2c\mathbf{i} + (7 - c)t\mathbf{j}$, as required

b $t = 5 \Rightarrow \mathbf{F} = 2c\mathbf{i} + (7 - c)5\mathbf{j}$

$|\mathbf{F}|^2 = 4c^2 + 25(7 - c)^2 = 17^2$

$4c^2 + 1225 - 350c + 25c^2 = 289$

$29c^2 - 350c + 936 = 0$

$(c - 4)(29c - 234) = 0$

$c = 4, \frac{234}{29} \approx 8.07$

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Exercise D, Question 13

Question:

A ball, attached to the end of an elastic string, is moving in a vertical line. The motion of the ball is modelled as a particle B moving along a vertical axis so that its displacement, x m, from a fixed point O on the line at time t seconds is given by $x = 0.6\cos\left(\frac{\pi t}{3}\right)$. Find

a the distance of B from O when $t = \frac{1}{2}$,

b the smallest positive value of t for which B is instantaneously at rest,

c the magnitude of the acceleration of B when $t = 1$. Give your answer to 3 significant figures.

Solution:

a When $t = \frac{1}{2}$

$$x = 0.6\cos\left(\frac{\pi}{3} \times \frac{1}{2}\right) = 0.6\cos\frac{\pi}{6}$$

$$= 0.6 \times \frac{\sqrt{3}}{2} = 0.3\sqrt{3}$$

The distance of B from O when $t = \frac{1}{2}$ is $0.3\sqrt{3}$ m .

b $v = \frac{dx}{dt} = -0.6 \times \frac{\pi}{3}\sin\left(\frac{\pi t}{3}\right)$

The smallest positive value at which $v = 0$ is given by

$$\frac{\pi t}{3} = \frac{\pi}{2} \Rightarrow t = \frac{3}{2}$$

c $a = \frac{dv}{dt} = -0.6\left(\frac{\pi}{3}\right)^2 \cos\left(\frac{\pi t}{3}\right)$

When $t = 1$

$$a = -0.6\left(\frac{\pi}{3}\right)^2 \cos\left(\frac{\pi}{3}\right) = -0.3289\ldots$$

The magnitude of the acceleration of B when $t = 1$ is 0.329 m s^{-2} (3 s.f.)

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Exercise D, Question 14

Question:

A light spot S moves along a straight line on a screen. At time $t = 0$, S is at a point O. At time t seconds (where $t \geq 0$) the distance, x cm, of S from O is given by $x = 4te^{-0.5t}$. Find

a the acceleration of S when $t = \ln 4$,

b the greatest distance of S from O.

Solution:

a $v = \frac{dx}{dt} = 4\,e^{-0.5t} - 2t\,e^{-0.5t}$

$a = \frac{dv}{dt} = -2\,e^{-0.5t} - 2\,e^{-0.5t} + t\,e^{-0.5t} = (t-4)e^{-0.5t}$

When $t = \ln 4$

$a = (\ln 4 - 4)e^{-0.5\ln 4} = (2\ln 2 - 4)e^{\ln\frac{1}{2}}$

$= \frac{1}{2}(2\ln 2 - 4) = \ln 2 - 2$

The acceleration of S when $t = \ln 4$ is $(\ln 2 - 2)$ m s^{-2} in the direction of x increasing.

b For a maximum of x, $\frac{dx}{dt} = v = 0$

$v = (4 - 2t)e^{-0.5t} = 0 \Rightarrow t = 2$

When $t = 2$

$x = 4 \times 2\,e^{-0.5\times 2} = 8\,e^{-1}$

The greatest distance of S from O is $\frac{8}{e}$ m.

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Exercise D, Question 15

Question:

A particle P is projected with velocity $(3u\mathbf{i}+4u\mathbf{j})$ m s^{-1} from a fixed point O on a horizontal plane. Given that P strikes the plane at a point 750 m from O.

a show that $u = 17.5$,

b calculate the greatest height above the plane reached by P,

c find the angle the direction of motion of P makes with $\mathbf{i}$ when $t = 5$.

Solution:

a Taking components horizontally and vertically

$\text{R}(\rightarrow)\quad u_x = 3u$

$\text{R}(\uparrow)\quad u_y = 4u$

$\text{R}(\rightarrow)\quad$ distance = speed × time

$$750 = 3ut \Rightarrow t = \frac{250}{u}$$

$\text{R}(\uparrow)\quad s = ut + \frac{1}{2}at^2$

$$0 = 4ut - 4.9t^2$$

$$0 = \frac{4u\times 250}{u} - 4.9\left(\frac{250}{u}\right)^2 = 1000 - \frac{306\,250}{u^2}$$

$$u^2 = \frac{306\,250}{1000} = 306.25$$

$$u = \sqrt{306.25} = 17.5, \text{ as required}$$

b

$$u_y = 4u = 4\times 17.5 = 70$$

$\text{R}(\uparrow)\quad v^2 = u^2 + 2as$

$$0^2 = 70^2 - 2\times 9.8\times s$$

$$s = \frac{70^2}{2\times 9.8} = 250$$

The greatest height above the plane reached by P is 250 m.

c When $t = 5$

$\text{R}(\uparrow)\quad v = u + at$

$$v_y = 70 - 9.8\times 5 = 21$$

$$\tan\theta = \frac{v_y}{u_x} = \frac{21}{3\times 17.5} = 0.4 \Rightarrow \theta = 21.8°$$

The angle the direction of motion of P makes with $\mathbf{i}$ when $t = 5$ is $22°$ (nearest degree).

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Exercise D, Question 16

Question:

A particle P is projected from a point on a horizontal plane with speed u at an angle of elevation θ.

a Show that the range of the projectile is $\frac{u^2 \sin 2\theta}{g}$.

b Hence find, as θ varies, the maximum range of the projectile.

c Given that the range of the projectile is $\frac{2u^2}{3g}$, find the two possible value of θ.

Give your answers to $0.1°$.

Solution:

a Taking components horizontally and vertically

$R(\rightarrow)\quad u_x = u\cos\theta$

$R(\uparrow)\quad u_y = u\sin\theta$

$R(\uparrow)\quad s = ut + \frac{1}{2}at^2$

$$0 = u\sin\theta t - \frac{1}{2}gt^2 = t\left(u\sin\theta - \frac{1}{2}gt\right)$$

$t \neq 0$

$$t = \frac{2u\sin\theta}{g}$$

Let the range be R

distance = speed × time

$$R = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{2u\sin\theta\cos\theta}{g}$$

Using the identity $\sin 2\theta = 2\sin\theta\cos\theta$

$$R = \frac{u^2\sin 2\theta}{g}$$

b R is a maximum when $\sin 2\theta = 1$, that is when $\theta = 45°$.

The maximum range of the projectile is $\frac{u^2}{g}$.

c If $R = \frac{2u^2}{3g}$, $\frac{2u^2}{3g} = \frac{u^2\sin 2\theta}{g}$

$$\sin 2\theta = \frac{2}{3}$$

$$2\theta = 41.81°, (180 - 41.81)°$$

$$\theta = 20.9°, 69.1°, \text{(nearest } 0.1°)$$

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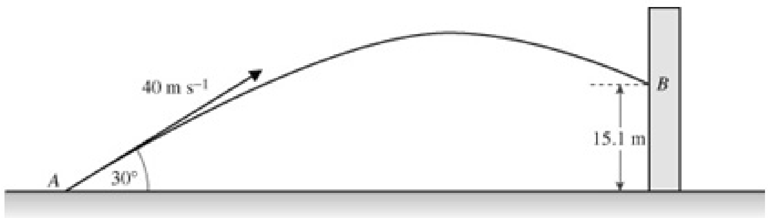
Exercise D, Question 17

Question:



A golf ball is driven from a point A with a speed of 40 m s^{-1} at an angle of elevation of 30°. On its downward flight, the ball hits an advertising hoarding at a height 15.1 m above the level of A, as shown in the diagram above. Find

a the time taken by the ball to reach its greatest height above A,

b the time taken by the ball to travel from A to B,

c the speed with which the ball hits the hoarding.

Solution:

Taking components horizontally and vertically

$\text{R}(\rightarrow) \quad u_x = 40\cos 30^\circ = 20\sqrt{3}$

$\text{R}(\uparrow) \quad u_y = 40\sin 30^\circ = 20$

a

$\text{R}(\uparrow) \quad v = u + at$

$$0 = 20 - 9.8t \Rightarrow t = \frac{20}{9.8} = 2.0408\ldots$$

The time taken by the ball to reach its greatest height above A is 2.0 s (2 s.f.)

b

$\text{R}(\uparrow) \; s = ut + \frac{1}{2}at^2$

$$15.1 = 20t - 4.9t^2$$

$$4.9t^2 - 20t + 15.1 = 0$$

$$(t-1)(4.9t - 15.1) = 0$$

On the way down the time must be greater than the result in part **a**, so $t \neq 1$.

$t = \frac{15.1}{4.9} = 3.0816\ldots$

The time taken for the ball to travel from A to B is 3.1 s (2 s.f.).

c

$\text{R}(\uparrow) \quad v_y = u + at$

$$= 20 - 9.8 \times \frac{15.1}{4.9} = -10.2$$

$$V^2 = u_x^2 + v_y^2 = (20\sqrt{3})^2 + (-10.2)^2 = 1304.04$$

$$V = \sqrt{1304.04} = 36.111\ldots$$

The speed with which the ball hits the hoarding is 36 m s^{-1} (2 s.f.).

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Exercise D, Question 18

Question:

A particle P passes through a point O and moves in a straight line. The displacement, s metres, of P from O, t seconds after passing through O is given by

$$s = -t^3 + 11t^2 - 24t$$

a Find an expression for the velocity, v m s^{-1}, of P at time t seconds.

b Calculate the values of t at which P is instantaneously at rest.

c Find the value of t at which the acceleration is zero.

d Sketch a velocity-time graph to illustrate the motion of P in the interval $0 \le t \le 6$, showing on your sketch the coordinates of the points at which the graph crosses the axes.

e Calculate the values of t in the interval $0 \le t \le 6$ between which the speed of P is greater than 16 m s^{-1}.

Solution:

a $v = \frac{ds}{dt} = -3t^2 + 22t - 24$

The velocity of P after t seconds is $(-3t^2 + 22t - 24)$ m s^{-1}.

b When $v = 0$

$3t^2 - 22t + 24 = (3t - 4)(t - 6) = 0$

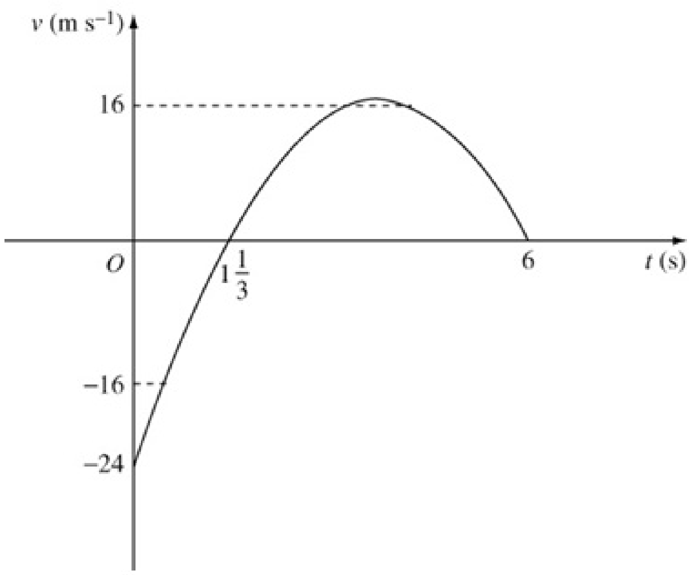
$t = \frac{4}{3}, 6$

c

$a = \frac{dv}{dt} = -6t + 22 = 0$

$t = \frac{22}{6} = \frac{11}{3}$

d



e The speed of P is 16 when $v = 16$ and $v = -16$.

When $v = 16$

$-3t^2 + 22t - 24 = 16$

$3t^2 - 22t + 40 = 0$

$(3t - 10)(t - 4) = 0$

$t = \frac{10}{3}, 4$

When $v = -16$

$-3t^2 + 22t - 24 = -16$

$3t^2 - 22t + 8 = 0$

$t = \frac{22 \pm \sqrt{(484 - 96)}}{6} = 0.38, 6.95 \text{ (2 d.p.)}$

From the diagram in part **d**, the required values are

$0 \le t < 0.38, \frac{10}{3} < t < 4$

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Exercise D, Question 19

Question:

A point P moves in a straight line so that, at time t seconds, its displacement from a fixed point O on the line is given by

$$s = \begin{cases} 4t^2, & 0 \le t \le 3 \\ 24t - 36, & 3 < t \le 6 \\ -252 + 96t - 6t^2, & t > 6. \end{cases}$$

Find

a the velocity of P when $t = 4$,

b the velocity of P when $t = 10$,

c the greatest positive displacement of P from O,

d the values of s when the speed of P is 18 m s^{-1}.

Solution:

a When $t = 4$, t is in the range $3 < t \leq 6$, so $s = 24t - 36$

$$v = \frac{ds}{dt} = 24$$

The velocity of P when $t = 4$ is 24 m s^{-1} in the direction of s increasing.

b When $t = 10$, t is in the range $t > 6$, so $s = -252 + 96t - 6t^2$

$$v = \frac{ds}{dt} = 96 - 12t$$

When $t = 10$

$$v = 96 - 12 \times 10 = -24$$

The velocity of P when $t = 10$ is 24 m s^{-1} in the direction of s decreasing.

c The maximum displacement is when $\frac{ds}{dt} = v = 0$

$$96 - 12t = 0 \Rightarrow t = 8$$

When $t = 8$

$$s = -252 + 96 \times 8 - 6 \times 8^2 = 132$$

The greatest positive displacement of P from O is 132 m.

d The speed of P is 18 m s^{-1} when $v = \pm 18$

In the range $0 \leq t \leq 3$

$$v = \frac{ds}{dt} = 8t = 18 \Rightarrow t = \frac{9}{4}$$

When $t = \frac{9}{4}, s = 4 \times \left(\frac{9}{4}\right)^2 = 20.25$

In the range $t > 6$

$$v = 96 - 12t = 18 \Rightarrow t = \frac{96-18}{12} = 6.5$$

$$s = -252 + 96 \times 6.5 - 6 \times 6.5^2 = 118.5$$

$$v = 96 - 12t = -18 \Rightarrow t = \frac{96+18}{12} = 9.5$$

$s = -252 + 96 \times 9.5 - 6 \times 9.5^2 = 118.5$, the same result as for $v = 18$

The values of s when the speed of P is 18 m s^{-1} are 20.25 and 118.5.

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Exercise D, Question 20

Question:

The position vector of a particle P, with respect to a fixed origin O, at time t seconds (where $t \geq 0$) is $\left[\left(6t - \frac{1}{2}t^3\right)\mathbf{i} + (3t^2 - 8t)\mathbf{j}\right]$ m . At time t seconds, the velocity of a second particle Q, moving in the same plane as P, is $(-8\mathbf{i} + 3t\mathbf{j})$ m s^{-1}.

a Find the value of t at the instant when the direction of motion of P is perpendicular to the direction of motion of Q.

b Given that P and Q collide when $t = 4$, find the position vector of Q with respect to O when $t = 0$.

Solution:

a For P

$$\mathbf{v} = \dot{\mathbf{p}} = \left(6 - \frac{3}{2}t\right)\mathbf{i} + (6t - 8)\mathbf{j}$$

The tangent the angle the direction of P makes with $\mathbf{i}$ is given by

$$m = \frac{6t - 8}{6 - \frac{3}{2}t}$$

The tangent the angle the direction of Q makes with $\mathbf{i}$ is given by

$$m' = -\frac{3t}{8}$$

Using $mm' = -1$

$$\frac{6t-8}{6-\frac{3}{2}t} \times -\frac{3}{8}t = -1$$

$$3t(6t - 8) = 8\left(6 - \frac{3}{2}t\right)$$

$$18t^2 - 24t = 48 - 12t$$

$$18t^2 - 12t - 48 = 0$$

$$3(t-2)(3t+4) = 0$$

$t \geq 0$

$t = 2$

b When $t = 4$

$$\mathbf{p} = \left(6 \times 24 - \frac{1}{2} \times 4^3\right)\mathbf{i} + \left(3 \times 4^2 - 8 \times 4\right)\mathbf{j} = -8\mathbf{i} + 16\mathbf{j}$$

For Q

$$\mathbf{q} = \int(-8\mathbf{i} + 3t\mathbf{j})\,dt = -8t\mathbf{i} + \frac{3}{2}t^2\mathbf{j} + \mathbf{A}$$

When $t = 4, \mathbf{p} = \mathbf{q} = -8\mathbf{i} + 16\mathbf{j}$

$$-8\mathbf{i} + 16\mathbf{j} = -32\mathbf{i} + 24\mathbf{j} + \mathbf{A} \Rightarrow \mathbf{A} = 24\mathbf{i} - 8\mathbf{j}$$

$$\mathbf{q} = (24 - 8t)\mathbf{i} + \left(\frac{3}{2}t^2 - 8\right)\mathbf{j}$$

When $t = 0$

$$\mathbf{q} = 24\mathbf{i} - 8\mathbf{j}$$

The position vector of Q with respect to O when $t = 0$ is $(24\mathbf{i} - 8\mathbf{j})$ m .